Short Course on Experimental Dynamic Substructuring

Module #9: Transmission Simulator Method

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Short Course Notes For:
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Component Mode Synthesis

- As discussed previously, if we have the modal parameters of two systems, either from analysis or experiment, we can write:

$$\begin{bmatrix} \mathbf{I}^{A} & 0 \\ 0 & \mathbf{I}^{B} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}^{A} \\ \ddot{\mathbf{q}}^{B} \end{Bmatrix} + \begin{bmatrix} \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]^{A} & \mathbf{0} \\ \mathbf{0} & \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]^{B} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}^{A} \\ \dot{\mathbf{q}}^{B} \end{Bmatrix} + \begin{bmatrix} \left[{}^{\backslash}\omega_r^{2}{}_{\backslash}\right]^{A} & \mathbf{0} \\ \mathbf{0} & \left[{}^{\backslash}\omega_r^{2}{}_{\backslash}\right]^{B} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{A} \\ \mathbf{q}^{B} \end{Bmatrix} =$$

$$= \begin{bmatrix} (\mathbf{\Phi}^{A})^{T} & \mathbf{0} \\ \mathbf{0} & (\mathbf{\Phi}^{B})^{T} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^{A} \\ \mathbf{F}^{B} \end{Bmatrix}$$

$$\begin{Bmatrix} \mathbf{y}^{A} \\ \mathbf{y}^{B} \end{Bmatrix} = \begin{bmatrix} \mathbf{\Phi}^{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{B} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{A} \\ \mathbf{q}^{B} \end{Bmatrix}$$

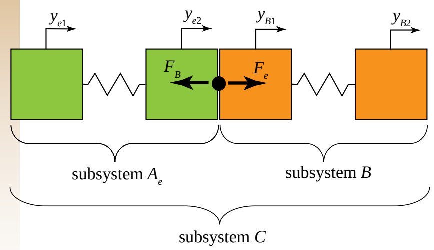
- Then, constraints in the following form are enforced to couple the substructures:

$$\mathbf{B}_{p} \begin{Bmatrix} \mathbf{y}^{A} \\ \mathbf{y}^{B} \end{Bmatrix} = \mathbf{0} \qquad \mathbf{B} = \mathbf{B}_{p} \begin{bmatrix} \mathbf{\Phi}^{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{B} \end{bmatrix} \qquad \mathbf{B} \begin{Bmatrix} \mathbf{q}^{A} \\ \mathbf{q}^{B} \end{Bmatrix} = \mathbf{0}$$

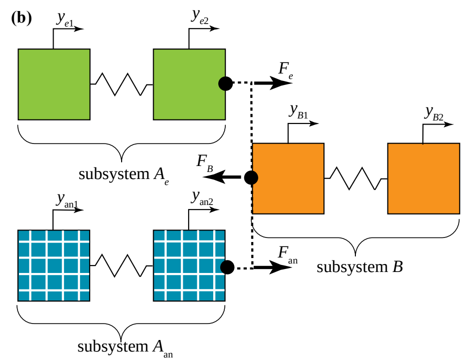
Substructure Uncoupling

- Our approach to "uncouple" structures works by constraining a model of one component to the structure that cancels the forces at the interface(s).

(a)



(b)



$$\mathbf{I}_e\ddot{\mathbf{q}}_e + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e \dot{\mathbf{q}}_e + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_e \mathbf{q}_e = \mathbf{\Phi}_e{}^{\mathrm{T}}\mathbf{F}_e$$

$$+\quad \mathbf{I}_{\mathrm{an}}\ddot{\mathbf{q}}_{\mathrm{an}} + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{\mathrm{an}} \dot{\mathbf{q}}_{\mathrm{an}} + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{\mathrm{an}} \mathbf{q}_{\mathrm{an}} = \mathbf{\Phi}_{\mathrm{an}}{}^{\mathrm{T}}\mathbf{F}_{\mathrm{an}}$$

$$0$$

The interfaces forces can be made to cancel if:

1.) The modal parameters of the analytical and true fixtures are equal and opposite, i.e. $\left[{}^{\backslash}\omega_{r\backslash}^2\right]_{\mathrm{an}} = -\left[{}^{\backslash}\omega_{r\backslash}^2\right]_e$ and similarly for the mass and damping terms.

2.) The motion of both the experimental and analytical fixtures is the same, $\{\mathbf{q}_{\mathrm{an}}(t)\} = \{\mathbf{q}_e(t)\}$

3.) The mode shapes of both the experimental and analytical models are equal $\mathbf{\Phi}_{\mathrm{an}} = \mathbf{\Phi}_e$.

4.) The interface force vector is in the range space of $\mathbf{\Phi}_{\mathrm{an}}{}^{\mathrm{T}}$.

$$\mathbf{I}_e\ddot{\mathbf{q}}_e + \mathbf{I}_{\mathrm{an}}\ddot{\mathbf{q}}_{\mathrm{an}} + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e \dot{\mathbf{q}}_e + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{\mathrm{an}} \dot{\mathbf{q}}_{\mathrm{an}} + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_e \mathbf{q}_e + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{\mathrm{an}} \mathbf{q}_{\mathrm{an}} =$$
$$= \mathbf{\Phi}_e{}^{\mathrm{T}}\mathbf{F}_e + \mathbf{\Phi}_{\mathrm{an}}{}^{\mathrm{T}}\mathbf{F}_{\mathrm{an}}$$

Connection Methods – CPT and MCFS

- One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
  - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:

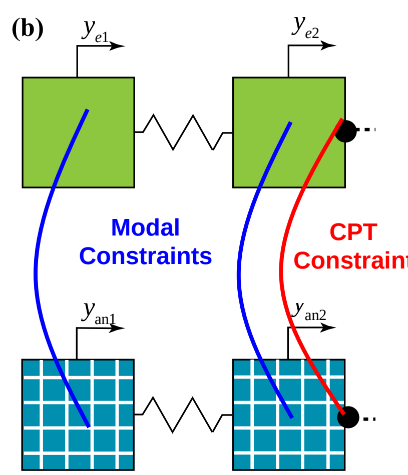
$$\begin{Bmatrix} \{\mathbf{y}^{\mathrm{C}}\}_m \\ \{\mathbf{y}^{\mathrm{C}}\}_c \end{Bmatrix} \approx \begin{bmatrix} [\boldsymbol{\Phi}^{\mathrm{A}}{}_m] \\ [\boldsymbol{\Phi}^{\mathrm{A}}{}_c] \end{bmatrix} \{\mathbf{q}^{\mathrm{C}}\} \rightarrow \{\mathbf{y}^{\mathrm{C}}\}_c = [\boldsymbol{\Phi}^{\mathrm{A}}{}_c][\boldsymbol{\Phi}^{\mathrm{A}}{}_m]^{\dagger} \{\mathbf{y}^{\mathrm{C}}\}_m$$

  - Modal Constraint Method: Constrain the modal DOF of the Fixture model to their approximation on C: (aka MCFS or “Modal Constraint for Fixture and Subsystem”)

$$\left.\begin{aligned} \{\mathbf{q}^{\mathrm{A}}\} &= [\boldsymbol{\Phi}^{\mathrm{A}}{}_m]^{\dagger} \{\mathbf{y}^{\mathrm{A}}\}_m \\ \{\mathbf{q}^{\mathrm{C}}\} &= [\boldsymbol{\Phi}^{\mathrm{A}}{}_m]^{\dagger} \{\mathbf{y}^{\mathrm{C}}\}_m \end{aligned}\right\} \rightarrow \{\mathbf{q}^{\mathrm{A}}\} = \{\mathbf{q}^{\mathrm{C}}\}$$

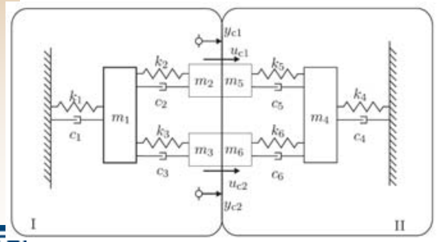
Connection Methods – Rationalization

- CMS can be very sensitive to errors when removing a substructure from a system.
- If the fixture is elastic, then it will probably have more modal DOF (N_A) than there are connection point DOF (N_c).
  - e.g. requiring equal 2D motion at the connection point enforces only (3) constraints.
  - When the number of constraints is N_c<N_A, it is unlikely that the modes of both structures will match and the interface forces most likely will not cancel!

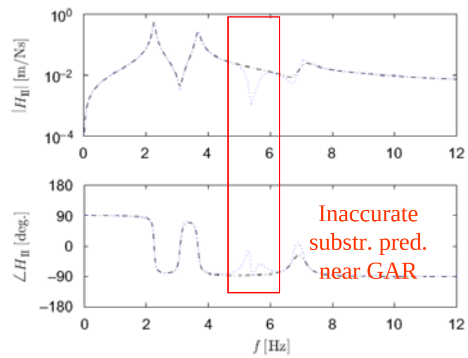


Related Techniques

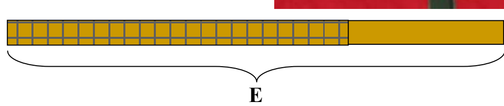
- Sjovall & Abrahamsson observed a similar phenomenon using FBS and called it "generalized anti-resonance" (GAR).
- Suggested using internal DOF to eliminate motions that are poorly observable on the interface DOF.
  - *W. D'Ambrogio and A. Fregolent, (IMAC XXVII) Orlando, Florida, 2009.*
  - *Sjovall, P. and T. Abrahamsson (2008). <u>MSSP</u> **22**(1): 15-33.*



$$H(\omega_{\mathrm{GAR}})U = 0 \text{ for } U \neq 0$$

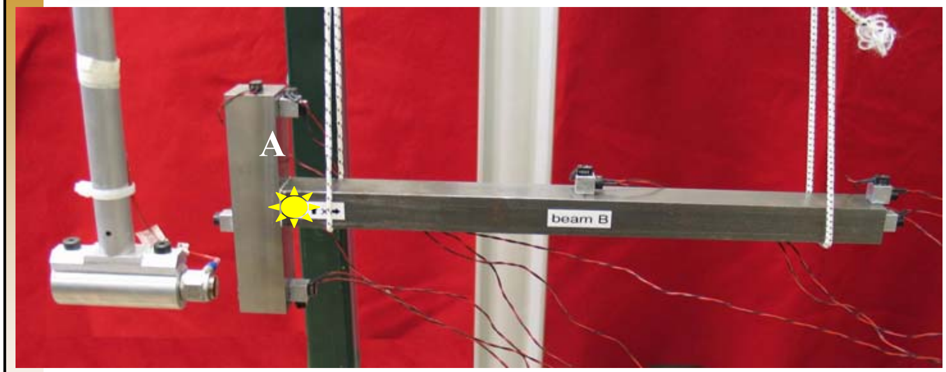


Test Case #1: T-Beam



- Objective: Join an experimental model of beam B to an analytical model for beam D at point y_c.
  - Measurements on system C = beam B + fixture A
  - Analytical model (Euler-Bernoulli) for fixture A removed from C
  - Beam B + Beam D (tuned Euler-Bernoulli analytical model) = assembly Beam E

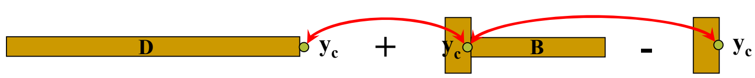
Experimental Procedure



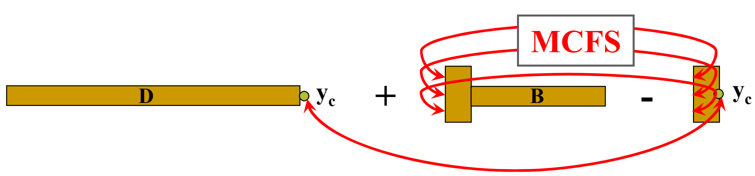
- Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.
- Hammer used to excite the largest bandwidth (largest number of modes) possible and to avoid modifying structure with the exciter.

Cases Considered

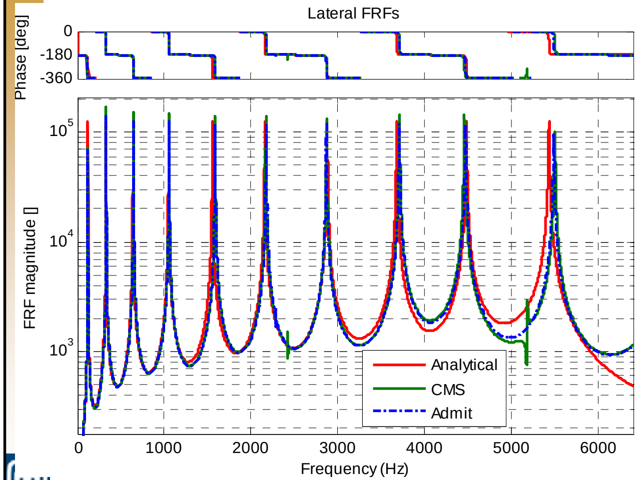
- Case 1: **CPT**: Models for A, C and D joined at the connection point.
  - Case 1a: Rigid Fixture Model
  - Case 1b: Elastic Fixture Model



- Case 2: **MCFS**: Models for A and C joined using MCFS method.
  - Elastic Fixture Model



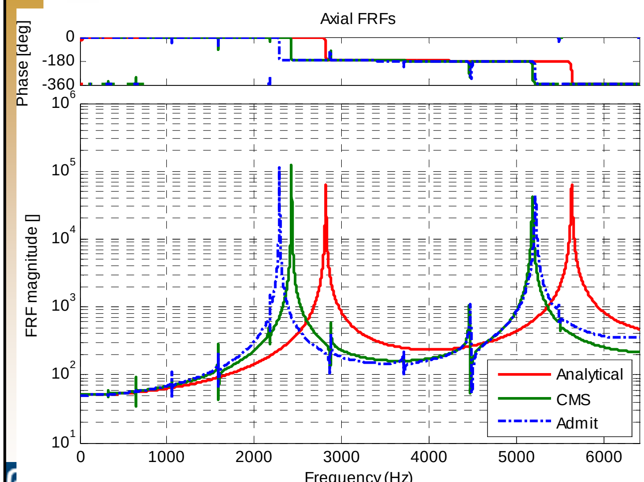
Case 1a: Rigid A, CPT



"Admit" = FBS with reconstructed FRFs

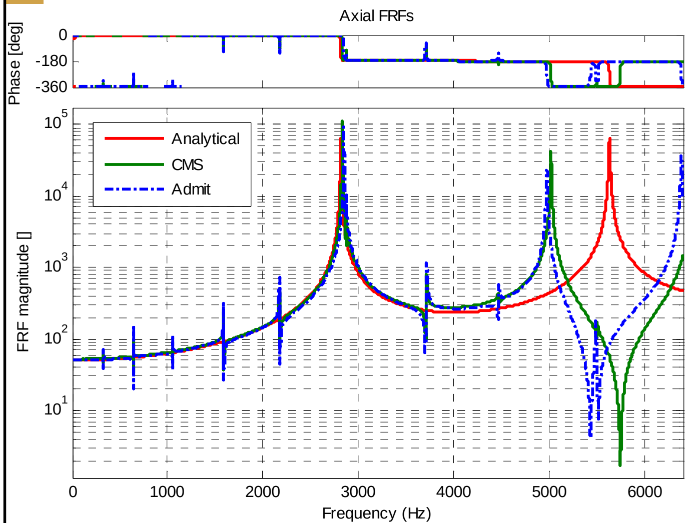
- Excellent results obtained in the lateral direction (Bending Modes).
- Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.

Case 1a: Rigid A, CPT



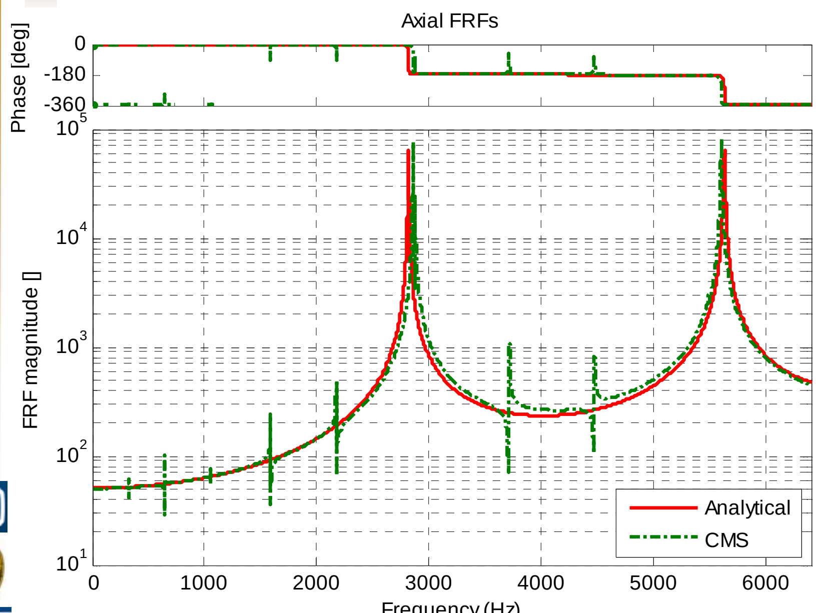
- Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
  - Fixture flexibility is important!
- Contamination at the bending natural frequencies.
  - Possibly due to small curve fitting errors or cross axis sensitivity.

Case 1b: Flexible A, CPT

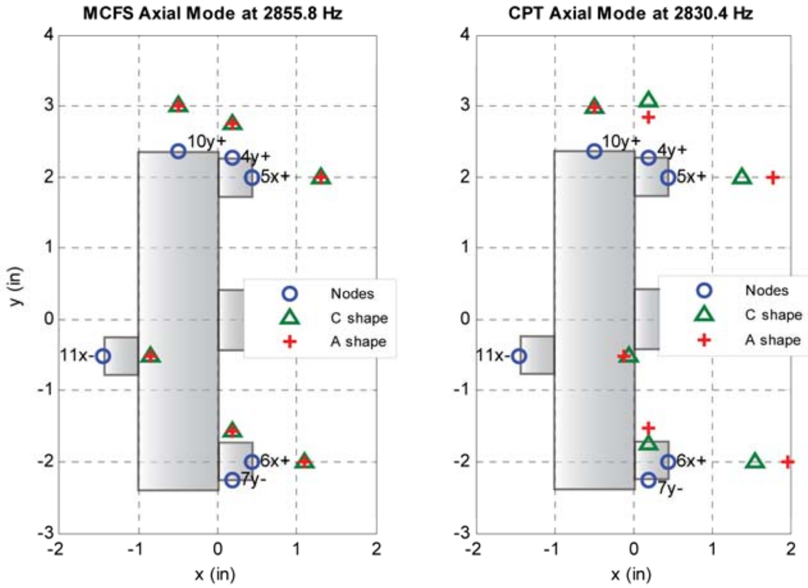


- Both FBS and CMS accurately predict the first axial frequency with the flexible fixture model.
- Both methods more severely under-predict the second axial mode.
- Both predict a spurious zero near 5500 Hz.
- The Lateral FRFs were similar to those shown previously.

Case 2: Flexible A, MCFS Method

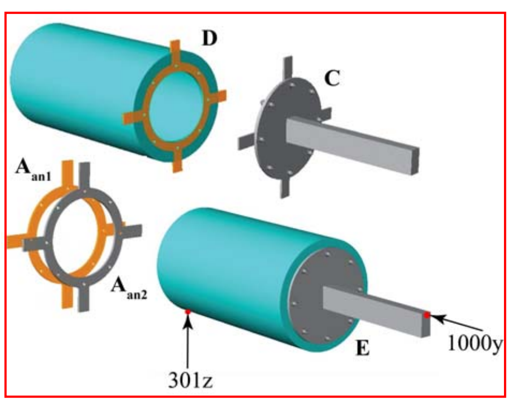


Explanation:



Significance:

- Now it is possible to robustly remove one flexible structure from another.
- The fixture modal basis gives a convenient basis for the interface – CAN BE USED TO REMOVE/COUPLE STRUCTURES THAT CONNECT ALONG A CONTINUOUS INTERFACE!



Transmission Simulator Method Enables use of Continuous Interfaces

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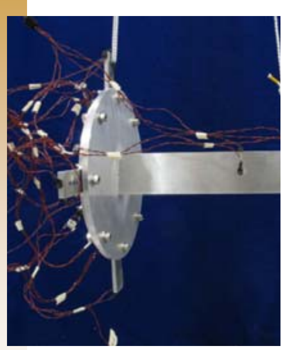
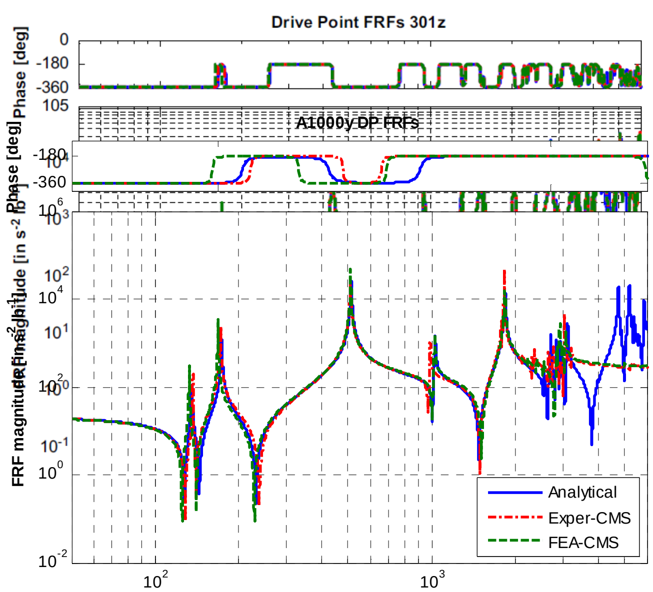




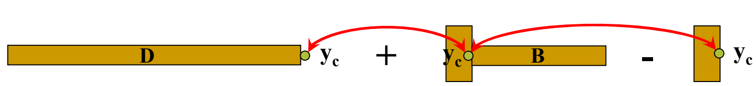
Transmission Simulator Design

- Design the TS so that it can be instrumented to capture all of its relevant dynamic modes so that $[\Phi_m]$ will be well conditioned for inversion.
- Choose the mass of the fixture to bring more modes into the testable bandwidth.
- Avoid designs that contain joints, intricate geometry, or other features that are difficult to accurately model analytically. Making the fixture of one piece, without any joints, goes a long way toward meeting this guideline.
- The joint between the fixture and the test article should replicate the actual joint in the system of interest as closely as possible, to capture the joint stiffness and damping.
- The fixture's impedance should roughly resemble that of the built-up structure. The fixture might just be a chopped off version of the other substructure which precisely mimics the mass and stiffness near the connection interface.

Problem of Negative Mass (& Stiffness)

- The transmission simulator must be removed to obtain the desired experimental model.
- The subtraction process may introduce negative mass and stiffness.
- Models with negative mass and stiffness are not readily imported into FEA packages, and they may introduce errors or counter-intuitive results.
- Example: Beam problem mentioned earlier

	Complex f_n (Hz)	eig(M) < 0
Case 1a Rigid Fixture, Con. Pt.	0 + i*8.9e-5 0 + i*5.5e5	-0.011
Case 1b Flexible Fixture, Con. Pt.	0 + i*1.36e-4 8951 – i*2450 8951 + i*2450	-1
Case 2 Flexible Fixture, MCFS	0 + i*2.28e-4 13050 – i*4285 13050 + i*4285	-0.086



Modal Scale Factor Method (Mayes)

- Negative mass implies that too much mass is being subtracted $\rightarrow$ the scale factors of the transmission simulator modes may be in error.
  - Solution: Adjust the modal scale factors to produce a positive-definite mass matrix. But how?
- Eigenvalues of Mass Matrix of B (structure of interest)

$$\mathbf{M}_{\mathrm{B}}\boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k \qquad \lambda_k < 0 \quad k = 0 \ldots N_{\mathrm{neg}}$$

- There are typically only a few negative eigenvalues and several modal scale factors (mm_j) that could be modified $\rightarrow$ under-constrained optimization problem.

$$\mathbf{f}\left(\mathrm{mm}_j\right) = \begin{bmatrix} \lambda_k & \cdots & \lambda_{N_{\mathrm{neg}}} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{f}(\mathrm{mm}_j + \Delta \mathrm{mm}_j) = \mathbf{f}(\mathrm{mm}_j) + \left[\nabla \mathbf{f}(\mathrm{mm}_j)\right]_{\mathrm{mm}_j} \Delta \mathrm{mm}_j$$

Added Mass Method (Allen)

- Negative mass can be eliminated by adding mass to the structure until the total mass is positive.
  - Suppose point masses are added at any of the points where the system's modes were measured. The resulting EOM are identical only with the mass matrix replaced by:

$$\mathbf{M}_{\mathrm{B}} + \Delta \mathbf{m}$$

  - The smallest change to the mass matrix that will make all of its eigenvalues positive is:

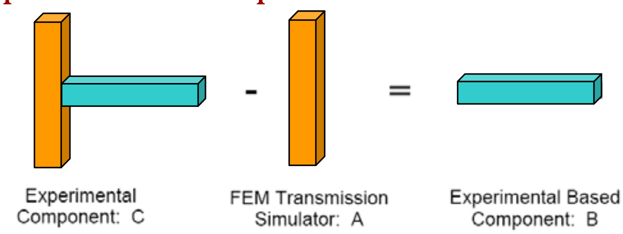
$$\Delta\lambda_k = \begin{cases} 0 & \lambda_k > 0 \\ -\lambda_k + \varepsilon & \lambda_k \le 0 \end{cases} \qquad \mathbf{M}_{\mathrm{B}}\boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k$$

  - where ε is a small value. This can be obtained using:

$$\Delta \mathbf{m} = \boldsymbol{\psi}\hat{\boldsymbol{\Lambda}}\boldsymbol{\psi}^{\mathrm{T}} \qquad \boldsymbol{\psi} = \begin{bmatrix} \boldsymbol{\psi}_k & \cdots & \boldsymbol{\psi}_{N_C} \end{bmatrix}$$

$$\hat{\boldsymbol{\Lambda}} = diag\left[\left(\lambda_1 + \Delta\lambda_1\right) \quad \cdots \quad \left(\lambda_{Nc} + \Delta\lambda_{N_C}\right)\right]$$

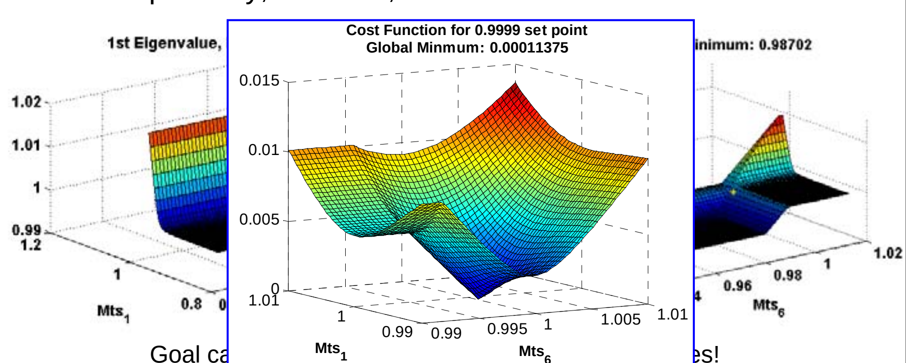
Application Examples



- Modal test obtains all modes below 20kHz for C and below 24kHz for A $\rightarrow$ 7 modes for TS and 15 modes for C
- The resulting model for B has two negative eigenvalues:
  - λ_1= -0.00050468
  - λ_2=-2.4058e-16, (essentially zero).
  - We desire to make these eigenvalues positive using the proposed methods.

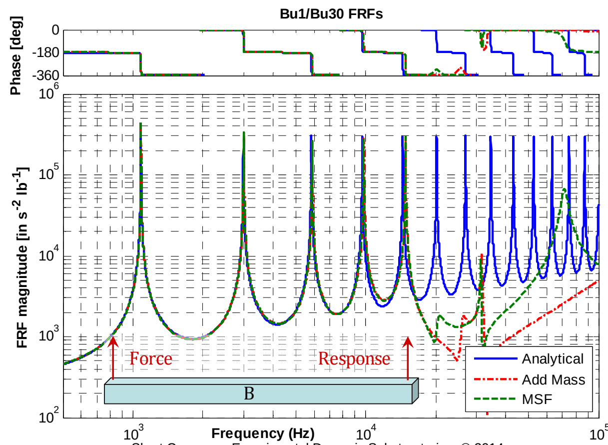
Results: Modal Scale Factor Method

- Jacobian (or metrics) reveal that the negative eigenvalues are most strongly affected by modes 1 and 6.
- Using ε=0.9999, after 3 iterations the algorithm converges on:
  - $M_{ts,1}$=0.99824 and $M_{ts,6}$=0.9999
  - (or increase mode scale factors by: 1.0008798 and 1.000050)
- Works perfectly, however, what if we chose ε=0.99?

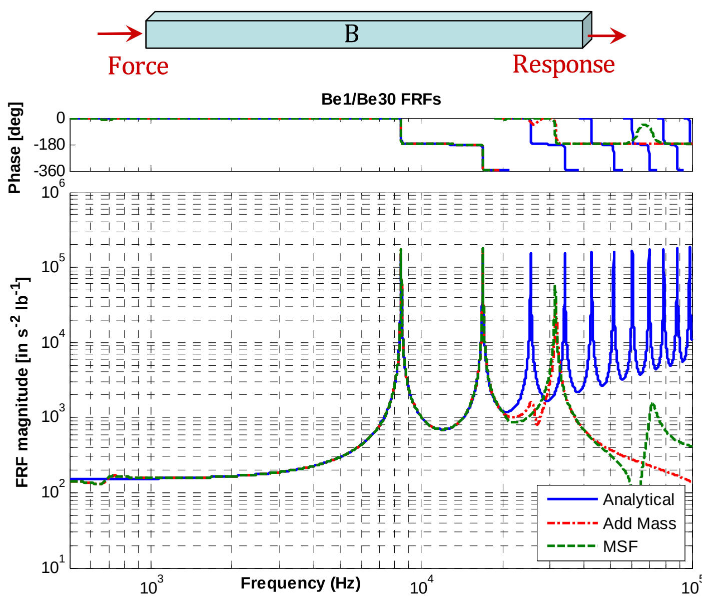


Results: Added Mass Method

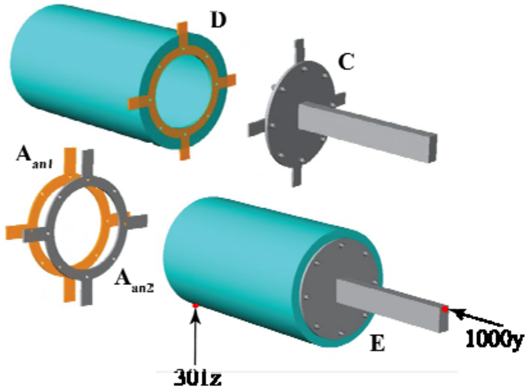
- Using ε=2.2e-14 (100 times larger than machine precision), smallest eigenvalue becomes 2.2e-14 as expected.
- How much mass had to be added to achieve this?
  - Ratio of the norm of $\mathbf{M_B}$ to $\Delta\mathbf{m}$ = 0.000505 (less than 0.05%!)

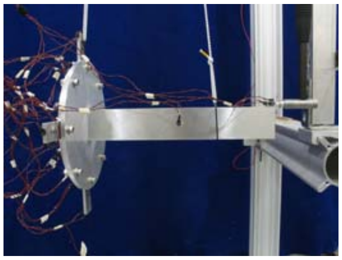


Axial FRFs: Both Methods



Three-Dimensional Plate-Beam System

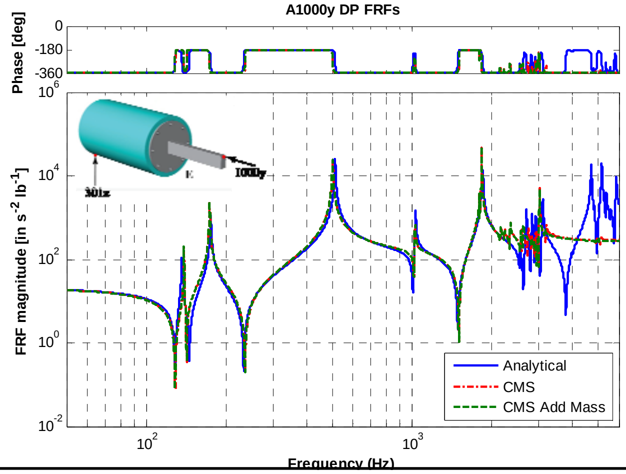




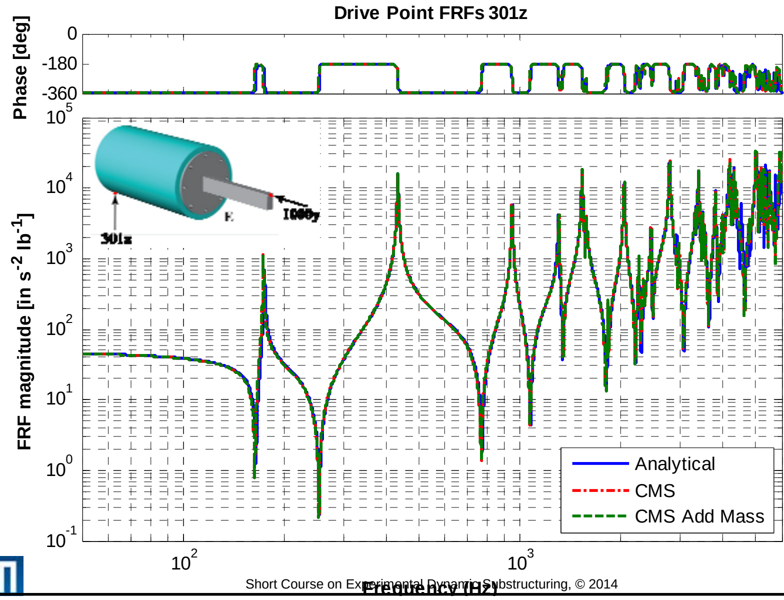
- Test performed to measure the modes of C
- Analytical system D modeled with finite elements with a second copy of the transmission simulator
- C & D are joined and then two copies of the transmission simulator are removed (using the FEA model of the TS)

Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A

- 4 Negative Eig.: λ_1= –0.197, λ_2= –0.0764, λ_3= –0.134, λ_4= –0.118
- Used added mass method, which revealed that 19.7% more mass had to be add to make the mass matrix positive definite.
- No deterioration observed in the quality of the FRFs

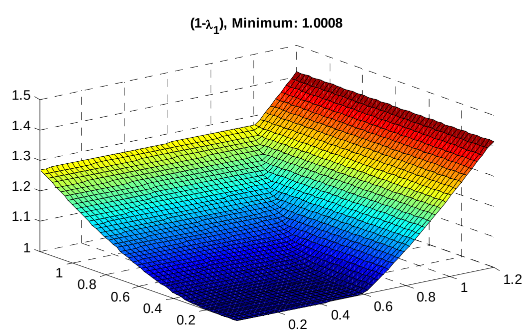


Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A



Case 2: Correct the mass matrix for C-A

- After removing the transmission simulator from C, the resulting model for B has two negative eigenvalues:
  - λ_1=-0.116 and λ_2=-0.0865
  - Using added mass method, the norm of the required mass addition is 11.6% of the norm of M_C
  - Mode Scale Method implemented with the Nelder-Mead Simplex algorithm using the dominant modes found by Kammer's τ metrics, the 9th and 13th modes.
    - Algorithm produced a negative modal mass, which was clearly not reasonable.



Conclusions

- Two methods have been proposed for correcting the negative mass that is sometimes obtained in substructuring predictions.
- One is highly unlikely to find a set of scale factors that produce positive mass using trial and error, so these new algorithms are a significant aid!
- Added Mass Method
  - Is straightforward to implement and was found to be successful in a variety of challenging cases.
  - Sometimes quite a large amount of mass had to be added.
- Modal Scale Factor Method:
  - This approach is more easily justified physically, since it avoids subtracting too much mass rather than correcting negative mass after the fact.
  - A nonlinear optimization problem must be solved to compute the modal scale factors of the transmission simulator that produce the desired result.

Additional Comments by R.L. Mayes